

## Waiting for the Revolution

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In the summer of 1900 Bertrand Russell attended the first World Congress of Philosophy in Paris. There he met the Italian mathematician Giuseppe Peano and learned of Peano's work in logic. In the fall of that year Russell extended Peano's monadic predicate calculus to a complete logic of relations. Russell was euphoric.

My sensations resembled those one has after climbing a mountain in a mist, when, on reaching the summit, the mist suddenly clears, and the country becomes visible for forty miles in every direction. For years I had been endeavoring to analyze the fundamental notions of mathematics, such as order and cardinal number. Suddenly, in the space of a few weeks, I discovered what appeared to be definitive answers to the problems which had baffled me for years. And in the course of discovering these answers, I was introducing a new mathematical technique, by which regions formerly abandoned to the vaguenesses of philosophers were conquered for the precision of exact formulae. Intellectually, the month of September 1900 was the highest point of my life.<sup>1</sup>

In 1905 Russell developed his theory of descriptions, the theory Frank Ramsey would later characterize as “that paradigm of philosophy”; and in 1910 the first volume of Russell and Whitehead's *Principia Mathematica* appeared.<sup>2</sup> Thus was begun the intellectual movement that would come to be known as analytic philosophy.

The fundamental idea of analytic philosophy, at least at first, was to assume that philosophical difficulties arise because we fail adequately to understand the notions on which those difficulties are based and then to resolve those difficulties by providing conceptual analyses of the problematic notions, analyses that would reveal their underlying logical form. Mathematics provided the model. Much as the confusions and difficulties surrounding, for example, the notion of a limit in calculus were resolved by the epsilon-delta definition due to Cauchy, Bolzano, and Weierstrass, so our philosophical confusions and difficulties were to be resolved by our coming to clarity about various non-mathematical concepts. But although analysis is clearly central to the practice of mathematics—because it is the means by which mathematicians develop the conceptions

they need to support rigorous reasoning in their proofs—the role of analysis in the practice of philosophy has turned out to be much less clear-cut. First, as eventually became apparent, philosophy has nothing like the complex system of checks and balances that exists in the practice of mathematics and ensures that one has significant, though not complete, cognitive control over putative developments. And this lack of any system of checks and balances in philosophy has led many philosophers to question whether intuition—by appeal to which, it is supposed, one assesses the fruits of one’s philosophical analysis—can have any role at all to play in the practice of philosophy. And if the success of any putative philosophical analysis *is* to be assessed by appeal to one’s intuitive sense of its success, then perhaps philosophers should investigate empirically, as (so-called) experimental philosophers do, what intuitions not only philosophers but also non-philosophers actually have in regard to this or that question of analysis.

A second problem the practice of analysis in philosophy has had to face—as Quine famously argues in “Two Dogmas of Empiricism”—is that it is not at all clear in the cases of concern to philosophy where meaning ends and fact begins. Because and insofar as the concepts of concern to philosophers are ineluctably caught up in the everyday world of which we speak, and are subject to the vicissitudes of our changing experiences, the traditional *a priori* methods of philosophy, including the method of philosophical analysis, have come to seem deeply problematic. And this has effected in turn just what Quine has urged, “a blurring of the supposed boundary between speculative metaphysics and natural science.”<sup>3</sup> The upshot, philosophy naturalized, is armchair science, philosophy as little more than speculative science.

The Quinean idea that there is no longer room for the practice of philosophy distinct from and in addition to the practice of empirical science has been hugely influential. And it has been so influential at least in part because it is reinforced by a conception of the history of philosophy and empirical science according to which the discipline of philosophy serves as little more than an incubator for the various other disciplines, other disciplines that, once they are sufficiently robust, can be separated off from philosophy to become autonomous fields of inquiry. Physics, for example, was at first a branch of philosophy but after the seventeenth century came to be contrasted with philosophy insofar as it is an experimental science. Psychology, the empirical study of the mind, and linguistics, the empirical study of language, similarly began life non-empirically, as branches of philosophy. And most recently the emergence of, first, cognitive science and then also neuroscience have seemed to many to have shown that philosophy has nothing left with which to concern itself. If consciousness, the last great mystery of the natural world, is now amenable to empirical investigation thanks to recent technological advances—for example, in brain imaging—then perhaps there really is nothing left for

philosophers to do. To this way of thinking, the sort of non-empirical inquiry that philosophers have traditionally engaged in had seemed viable as a form of intellectual inquiry only because, and so long as, we did not yet have the resources that are needed to engage in properly empirical investigations into the relevant phenomena. Because all questions have been revealed to be empirical questions, save, perhaps, for properly mathematical ones, there is nothing left for the philosopher to do. Philosophy no longer has any place in the intellectual culture.

Of course there are philosophers who disagree. Indeed, there are those who do not merely reject the idea that the empirical sciences can and should take over whatever questions remain in philosophy; they hold that the empirical sciences have nothing at all to contribute to their philosophical work. This is true, for example, of those analytic metaphysicians who develop theories that have in principle no empirical significance or consequences. It is also true of at least some neo-Aristotelians aiming to recover the ancient Greek conception of ourselves as rational animals in the world that was eclipsed by the rise of modern science. Both philosophical movements have been criticized, the first as an intellectually irresponsible game of wits to no purpose, the second as an intellectually naïve attempt to pretend that modernity never happened, that the rise of modern science can be ignored. There is surely something right in these criticisms: philosophers cannot simply turn their backs on the extraordinary developments in the sciences that have transformed not only our everyday lives but our very conception of the universe. (For the record, I am nonetheless deeply sympathetic to the spirit of the neo-Aristotelian movement.) Suppose, then, that we do take the full measure of the practice of modern science. Is there, even so, a way forward for philosophy?

Russell thought he had discovered “a new mathematical technique” that would enable philosophy finally to embark on the sure path of a science. Russell was wrong. Philosophy is not a science—not even, *pace* Kant, an *a priori* science alongside mathematics. Let us, for the purposes of argument, say that it is, as Bernard Williams characterizes it, a “general attempt to make the best sense of our life, and so of our intellectual activities, in the situation in which we find ourselves.”<sup>4</sup> The philosopher, in other words, aims not so much to know as to understand: to understand how it all hangs together, what it all means, whether it is in fact true, and why we should even care. And because philosophy aims in this way to understand, it cannot in the nature of things be replaced or superseded by the empirical sciences. Its questions are not empirical questions. They are questions that will remain even after all the empirical questions have been answered. Simply put, we need both, both philosophy and the empirical sciences. The problem is to understand how, exactly, this is to work.

After more than a century of the efforts of our ablest philosophers, the practice of analytic philosophy has failed to deliver of its promise of clarity and insight. Despite an impressive array of technical results, the major problems of philosophy have remained essentially untouched. And nowhere is this more evident than in the philosophy of mathematics. Over the course of the twentieth century, the philosophy of mathematics came to be so detached from, and so completely irrelevant to, mathematical practice that practicing mathematicians are no longer willing even to talk to philosophers if they can possibly avoid it. In response, a small group of philosophers of mathematics has emerged calling for a radically new approach in the philosophy of mathematics, an approach one central feature of which is the wholesale rejection of logic as a tool for understanding mathematical practice. *This is incredible*. How could it possibly be that *logic*, the concern of which is reasoning, is irrelevant to our understanding of mathematical practice, at the core of which is *reasoning*? Surprisingly, the question has an answer.

Although Frege is often cited as the father of modern logic, modern logic would have developed pretty much as it did had Frege never existed.<sup>5</sup> And as I have argued, in fact Frege was doing something quite different from what Russell and the analytic tradition following him took him to be doing.<sup>6</sup> Beginning with Russell, Frege's strange two-dimensional notation was systematically misread as a notation of the logic bequeathed to us by Russell, and because it was, Frege's truly revolutionary work in logic remains to this day almost completely unknown. Fortunately, the general idea of Frege's logical language, as it contrasts with our standard logical languages, is evident already in the positional system of Arabic numeration as that system contrasts with the system of Roman numeration. Reflecting on that latter system thus enables us to gain some insight into the former.

Roman numeration is a system of written signs that was devised for recording how many things there are in a collection. It is based on a primitive tally system in which one makes one mark for each thing in the collection resulting in a collection of marks that is in a one-to-one correspondence with the collection of things. The difference between such a tally system and the system of Roman numeration is only that in Roman numeration abbreviations are introduced: 'V' for a collection of five things, 'X' for a collection of ten things, and so on. Collections of such marks are then read additively: 'XVII', for example, is ten and five and one and one, that is, seventeen. (It is a late modification, and one that will not concern us here, to write, say, IX instead of VIIII, for nine, and so to distinguish IX, that is, nine, from XI, eleven.) In such a system, each sign means what it means independent of any context of use. The sign 'V' inevitably stands in for five things in the system of Roman numeration, 'X' for ten things and so on; it is merely a convention to

write the letters in a particular order. One could as easily write, for example, seventeen, XVII, as, say, VIXI.

Arabic numeration is more interesting insofar as although one *can* read it as a system very like that of Roman numeration, one can also read it differently, as a radically different *kind* of numeration system. Consider, for example, the numeral '647' of the positional system of Arabic numeration, and suppose, first, that just as in the case of Roman numeration each primitive sign of this system has its meaning independent of any context of use. The digit '6', for example, just is the numeral '6' on this reading; it is a name for the number six. And so for the other digits: each just is the relevant numeral, a name for some particular number, zero, or one, or two, and so on up to nine. What the *position* of the numeral tells one, on this reading, is what it is that one is counting, whether units, or tens, or hundreds, and so on. The position of the '6' in '647' tells us that it is counting hundreds: there are six hundreds. The position of the '4' tells us that it is counting tens: there are four tens. And the position of the '7' tells us that it is counting units: there are seven units. The whole is then to be read additively: there are six hundreds and four tens and seven units. On this reading, the system of Arabic numeration is essentially similar to Roman numeration. Obviously in this case, the order of the numerals matters because that is what is telling one what it is that is being counted. Nevertheless, the overall idea of the system is the same as that of Roman numeration: the collection of signs serves to record how many in a collection of things.

But we can also read the system of Arabic numeration differently. On the alternative reading we take the individual digits, '0' through '9', to function as numerals, as names for numbers, *only within a context of use*. Independent of any context of use, the primitive signs of the language, the individual digits, only express what we can call Fregean senses; independent of a context of use the primitive signs do not designate or mean or name anything. Now we put some primitive signs together to form a complex sign for some number. We take the digits '6' and '4' and '7', for example, to form the complex sign, the numeral, '647', which is a name for the (one) number six hundred and forty-seven. In this way, the digits do not function independently as numerals, as names for numbers; it is only in combinations that collections of the primitive signs (in the limit, a "combination" of only one digit) function as numerals, as names for numbers. Because, on this reading of the language, the primitive signs only express Fregean senses independent of any context of use, because they do not designate except in a context of use, complexes of those signs such as '647', although they do designate some one number (here, the number six hundred and forty-seven), do so through complex Fregean senses, senses that can serve as the basis for mathematical calculations precisely because they are complex. Because there are rules governing the manipulations of the primitive signs that together form complex signs for

numbers, reasoning according to those rules can reveal truths about the numbers that are designated by the relevant complex expressions.

Roman numeration serves merely to picture collections of things. One cannot reason mathematically in such a language; one can only manipulate the signs mechanically, as one might manipulate the things collected, for instance, combining them or dividing them up into smaller collections. The positional system of Arabic numeration, by contrast, though it *can* be read merely mechanically, as essentially like the system of Roman numeration, can *also* be read as a properly mathematical language. It can be read, that is, as exhibiting the contents of mathematical ideas, *what it is* to be, say, six hundred and forty-seven, and exhibiting that content in a mathematically tractable way, in a way enabling rigorous reasoning *in* the system of signs. Exactly this distinction applies also to different logical languages, in particular to our standard logical language as it contrasts with the language Frege devised in his 1879 logic. The logic that is bequeathed to us by Peano, Peirce, Russell, and others is a language the primitive signs of which function just as the signs of Roman numeration do, to designate independent of a context of use. The language serves to record or picture states of affairs, truth conditions. Frege's logic is different. It is a language within which the primitive signs only express senses independent of any context of use, and hence can be combined to form complex names for concepts on the basis of which to reason. What we need if we are to make real progress in philosophy is just such a Fregean logic because, as I show in *Realizing Reason*, it is this logic, as contrasted with standard logic, that radically transforms our understanding of the space of possibilities within which our thought can move.

If analytic philosophy is to become the vibrant and productive discipline it once held out promise of being, we need to begin anew with Frege's logic. We need to revisit developments in the nineteenth century, particularly in the practice of mathematics, so as to come to a better understanding of the great intellectual advances that were made in that century, and we need to recognize that those advances offer a new way forward, one we have hitherto failed to consider. It is in just this way that we will come to see that Frege's logic is something essentially and radically new, in just this way that we will come to see that, unlike standard logic, Frege's logic provides rich and powerful resources for addressing the traditional problems of philosophy, problems about, for example, our capacity for knowledge of the world around us, about the relationship of mind and body, and about the reality of objective values.<sup>7</sup>

We also need sharply to distinguish between mathematical languages—paradigmatically, the diagrammatic language of Euclidean geometry, the symbolic language of arithmetic and algebra, and Frege's logical language *Begriffsschrift*—and the logics that govern them, on the one hand, and everyday language and reasoning (including

philosophical reasoning), on the other. We need to stop trying to apply standards of rigor and perspicuity that are appropriate to mathematics also to everyday life—including the everyday life of mathematicians—and to our attempts to achieve an adequate philosophical understanding of that life. And we need explicitly to recognize that the empirical sciences are neither the arbiter of all things nor merely something we do. We must take the full measure of the ways in which the rise of modern science has been transformative of our self-understanding while at the same time recognizing that the sciences are not and cannot be the wellspring of our deepest self-understanding. And we need, finally, to return to the history of philosophy, not because the answers we seek are there but because Hegel was right: we cannot understand where we are and need to be until and unless we understand how we came to be here.

Philosophy is not a science among sciences. It has always been and at its best remains still today a rogue discipline within which anything can be called into question as reason sees fit, a discipline that has no particular subject matter and no particular method, a discipline powered only by a relentless, resolute, and passionate desire *to understand*. And there will always be practitioners of this discipline, thinkers who are conversant with the works of the great philosophers of the past, thinkers who are, in equal measure, intellectually curious and intellectually serious, thinkers who are willing to take risks and to question received wisdom and entrenched values. There are such thinkers, such philosophers, still today. Many of them one knows; some one does not.

But one will.

Come the revolution, one will.

## Notes

1. Russell 1975, 147.
2. Russell published his theory of descriptions in Russell 1905. Ramsey makes his remark about that theory in “Philosophy” (1929; repr. 1990). Cambridge University Press serially published the three volumes of *Principia Mathematica* in 1910, 1912, and 1913.
3. Quine 1951, 20.
4. Williams 2000, 479.
5. Both Hilary Putnam and W. V. O. Quine make this point, Putnam in Putnam 1982, and Quine in Quine 1995.

6. See Macbeth 2005 and 2014.
7. Again, see Macbeth 2014 for an extended elaboration and defense.

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